Optimal Insurance Design under Narrow Framing*

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Abstract

In this paper, we study insurance decisions when the policyholder evaluates insurance with narrow framing. We show that due to aversion to risk on the net insurance payoff, i.e., insurance indemnity minus insurance premium, narrow framing reduces insurance demand. This helps explaining the observed low insurance demand in many insurance markets. We also show that the optimal insurance contract involves a deductible and the coinsurance of losses above the deductible when transaction costs depend on the actuarial value of the policy. Moreover, when the policyholder is loss averse over the net insurance payoff, a fixed indemnity equal to insurance premium should be paid for a range of intermediate losses.

Keywords: Deductible, Hedging, Risk Sharing, Narrow Framing, Loss Aversion

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1 Introduction

A major puzzle in insurance economics is the fact that people often insure themselves too little. It is well documented that many people do not take out disaster insurance and health insurance policies even though premiums for such insurance contracts are heavily subsidized (Kunreuther et al., 1978; Brown and Finkelstein, 2011). In 2010, 50 million people in the US lacked health insurance (DeNavas-Walt et al., 2011). This failure to take out health insurance poses serious health risks. In particular, uninsured people are more likely to delay needed medical care or not receive it at all due to high costs (National Center for Health Statistics Data Brief, 2012). A well-known result from Mossin (1968) shows that partial insurance comes from a positive loading factor under standard expected utility theory. Thus, these phenomena can be explained only if insurance pricing is unrealistically actuarially unfair.

Standard factors such as transaction costs only partially explain the observed lack of coverage. For instance, Brown and Finkelstein (2007, 2011) showed that even though insurance loads are unusually large in the long-term care insurance market, they are not large enough to explain the small size of the market.[1] Alternative explanations such as informational factors were shown to be not satisfying either. Recently, Boyer et al. (2020) found that information constraints in the long-term care insurance market play a large role but there is limited scope for take-up to reach levels beyond 30% in their counterfactual analysis when these constraints are removed.

Findings from psychology and behavioral economics have raised the possible role of deviations from perfectly rational behavior in insurance purchases. A plausible explanation for this observation is that people may have some other motives when purchasing insurance, in addition to hedging the risk to wealth.[2] More specifically, insurance itself may be partially perceived as a risky investment due to narrow framing (e.g., Kahneman and Lovallo, 1993; Rabin and Weizsäcker, 2009), i.e., a certain loss of premium traded for

[1] High insurance loads could also be because there are more market frictions such as adverse selection (Akerlof, 1978) and moral hazard (Holmstrom, 1979) in this type of insurance markets.

[2] Regret theory may also help to explain the observed low insurance demand but only when the loading factor is high (see Braun and Muermann, 2004). However, it is difficult to justify low insurance demand when insurance is heavily subsidized. Other alternative explanations exist in the literature. For instance, Peter and Ying (2019) showed that when contract non-performance risk is perceived as ambiguous, ambiguity averse policyholders reduce insurance demand.

an uncertain gain from coverage. As noted by Giesbert et al. (2011) and Kunreuther et al. (2013), individuals often view insurance as a “poor investment”. If this is the case, the hedging effect that insurance will bring becomes undervalued. Recently, Gottlieb and Mitchell (2019) found in a survey study that respondents subject to narrow framing were significantly less likely to buy long-term care insurance than those who were not subject to narrow framing.[3] Moreover, whether or not an individual was subject to narrow framing explained more variation in insurance uptake than risk aversion or adverse selection.

In this paper, we assume that the policyholder holds mixed views with regard to insurance purchases: hedging the risk to wealth and gambling with insurance companies. The gambling motive is the only departure from the standard expected utility framework in our model. It captures the idea that people may be subject to a certain degree of narrow framing and care about the outcome of an insurance contract when it is viewed in isolation (Barberis and Huang, 2001, 2009). If the policyholder is averse to the risk in the net payoff of insurance, an insurance contract itself can be considered as an unfavorable gamble. We show that when a coinsurance policy is offered, the policyholder will prefer not to be fully insured even with a zero-premium loading. Moreover, if the degree of narrow framing is high enough, no insurance will be purchased. These results are in line with the existing strand of the literature on narrow framing and the observed low insurance take-up rates (e.g., Gottlieb and Smetters, 2016 and references herein). Contrary to previous studies, we work with a general loss distribution (rather than with binary risks). Moreover, when analyzing the optimal insurance demand, we do not assume loss aversion (Kahneman and Tversky, 1979). Our approach is therefore instrumental in analyzing the pure impact of narrow framing on insurance decisions, which is the usual case in real-world circumstances.

The main contribution we make to the literature is however to examine the problem of the optimal design of insurance contracts under narrow framing and linear transaction costs. Arrow (1971, 1974) was the first to examine this problem in the standard expected utility framework. He showed that the optimal contract contains a straight deductible when transaction costs are proportional to the indemnity. For losses below the deductible, the insurer pays no indemnity. For losses above the deductible, the indemnity equals the

[3] See also Gottlieb (2012) in which the author examined dynamic insurance demand through prospect theory and narrow framing.

loss minus the deductible. The intuition of this result is simple: the deductible insurance contract is the best compromise between the willingness to reduce risk and the need to limit the insurance deadweight cost. Any increment of indemnification opportunity should be used to cover the largest uncovered loss. This is because, under risk aversion, the marginal utility of wealth decreases.[4] Later work extended Arrow's results in other directions under the expected utility hypothesis. But few authors have addressed the issue of contracting with a behavioral agent.[5] As it turns out, straight deductibles are no longer optimal in the presence of a positive degree of narrow framing.

More explicitly, we show that the optimal contract contains a coinsurance rule above the deductible. The main idea behind this result is that at optimum the net payoff of insurance should not be too risky when insurance is evaluated in isolation. But this will of course reduce its hedging effect when insurance is viewed broadly. The optimal design of insurance contracts under narrow framing trade-offs between the gambling and hedging motives. Interestingly, these contractual features have appeared in different settings in the insurance literature. For instance, Raviv (1979) showed that coinsurance is optimal beyond a deductible level if cost functions depend on the size of insurance payments and are convex or if the insurer is risk averse. It has also been shown that due to costly state falsification, coinsurance above a deductible can be optimal in the presence of insurance fraud (Picard, 1996; Bond and Crocker, 1997; Crocker and Morgan, 1998; Fagart and Picard, 1999).[6] Here, we show that it can also be optimal when the policyholder views insurance contracts narrowly.

We further extend our study to allow for possible loss aversion over the net payoff of insurance, as is documented by Kahneman and Tversky (1979) and by many subsequent experimental findings such as Abdellaoui et al. (2007). When indemnities exceed the insurance premium, the policyholder with narrow framing makes gains; otherwise, he makes losses. Psychologically, losses may loom larger than gains. In this paper, we

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[4] Gollier and Schlesinger (1996) showed that any contract other than those with a straight deductible is dominated in the sense of second-order stochastic dominance (SSD) by the contract with a straight deductible with the same premium.

[5] One exception was by Gollier (2014) who examined the design of optimal insurance for an ambiguity averse policyholder. See a critical survey on this new topic in Köszegi (2014).

[6] This was initially raised as a natural assumption for most insurance contracts which guarantees that the policyholder truly states the incurred loss in case the policyholder cannot credibly commit to report honestly (Myerson, 1979).

assume that the net payoff of insurance is evaluated by a piece-wise linear utility function with a kink at the break-even point. We show that the optimal insurance contract contains a deductible for both low and high losses, while including a fixed indemnity equal to the premium for a certain range of intermediate losses.[7]

The remainder of this paper is organized as follows. Section 2 introduces the insurance demand model under narrow framing; Section 3 discusses the optimal insurance demand in a general setting when a coinsurance policy is offered; Section 4 characterizes the optimal insurance design under narrow framing; Section 5 addresses some possible concerns and concludes.

2 Insurance Model Under Narrow Framing

Our insurance model has two dates with $t = 0, 1$. At $t = 0$, the policyholder with an initial wealth w_0 is confronted by a potential loss x which will materialize at $t = 1$. Under limited liability, we restrict the loss x within an interval $[0, w_0]$.[8] Moreover, the loss could occur with a mass of probability at some points on the support on which its cumulative distribution function $F(x)$ is defined.

Before the loss occurs, the policyholder can insure himself by purchasing insurance. An insurance contract consists of two main components: the insurance premium P charged by the insurer and the insurance schedule $I(\cdot)$, with $I(x)$ denoting the indemnity that is paid by the insurer if the observable loss is x. For the sake of simplicity, an insurance contract can be represented by a pair $(I(\cdot), P)$ where in general, $I(x) \geq 0$ for all non-negative x.

We assume that the insurance market is perfectly competitive. Furthermore, we assume that the indemnity schedule $I(\cdot)$ is associated to an insurance premium P such that:

$$P = (1 + \tau) \int I(x) dF(x) \tag{2.1}$$

The integral in the above equation is the actuarial value of the policy. In other words, for each dollar of indemnity, the insurer incurs a cost $(1 + \tau)$ with τ being a non-negative

[7] We also note that there exist fixed-indemnity insurance plans, which are a type of supplemental health plan that gives the policyholder a fixed cash benefit payout in case he experiences specific illnesses or injuries covered by his policy.

[8] Without specification, the integrals throughout this paper are taken over this interval.

loading factor. This implies that when $\tau = 0$, the insurance premium is actuarially fair and that when τ is positive, it is actuarially unfair. Notice that this assumption is consistent with a risk-neutral insurer in a perfectly competitive insurance market with transactional costs but no entry costs. The results presented in this paper can easily be extended to the case where the premium is based upon the expected indemnity, i.e., $P = C(E(I(x)))$ where $C(\cdot)$ is a cost function with $C(0) = 0$ and $C'(\cdot) \geq 1$. When $C'(\cdot)$ is larger than 1, a marginal increase in coverage is costly since the increase in premium would be larger than the increase in the expected indemnity. This corresponds to the situation where τ is strictly positive in the paper. Since no further insight can be gained from such generalization and for ease of presentation, we simply assume linear transaction costs. Finally, our setting does not include any information asymmetries which would lead to moral hazard or adverse selection problems.

In most of the literature on insurance (e.g., Mossin, 1968; Smith, 1968; Schlesinger, 1981), the policyholder is assumed to have vNM expected utility functions defined on his final wealth or consumption. Such utility functions make a precise prediction as to how the policyholder evaluates an insurance contract he is offered: he merges the net payoff of the insurance (i.e., $I(x) - P$)) with the risk (i.e., $w_0 - x$) he is already facing to determine its effect on the distribution of his future wealth or consumption, and then checks if the new distribution is an improvement. In economic terms, insurance allows a risk averse individual to transfer his wealth from states of low marginal wealth utility to states of high marginal wealth utility.

In this paper, we assume that an insurance contract can be viewed in isolation by the policyholder as a gamble against the insurer, i.e., a certain loss of premium traded for an uncertain gain from coverage. The net amount of money the policyholder receives after buying an insurance contract is equal to $I(x) - P$. Therefore, the agent wins money from insurance companies when $I(x) - P$ is positive but loses money when it is negative. This argument is consistent with experimental findings that people are subject to narrow framing (Kahneman and Lovallo, 1993; Kahneman, 2003; Rabin and Weizsäcker, 2009). More formally, narrow framing means that, when an individual is deciding whether to accept a gamble, he uses a utility function that depends directly on the outcome of the gamble, not just indirectly via the gamble's contribution to his total wealth.

To be more realistic, we assume that the policyholder has mixed views regarding insurance purchase and is interested in maximizing both expected utility in his final wealth, i.e., traditional "broad" framing, and expected utility in the net payoff of insurance. The objective function of the policyholder is hereafter defined as follows:[9]

$$V(I(\cdot), P) = \int [u(w_0 - x + I(x) - P) + kg(I(x) - P)] \, dF(x) \tag{2.2}$$

The first term of the integrand represents wealth utility measured by a continuous and twice differentiable function u with $u'(\cdot) > 0$ and $u''(\cdot) < 0$, meaning that the policyholder prefers more wealth than less and dislikes any mean preserving spread on wealth in the sense of Rothschild and Stiglitz (1970). The second term of the integrand measures extra utility or disutility from the net payoff of insurance due to narrow framing. We make the following two assumptions:

(i) $g(0) = 0$;

(ii) $g(\cdot)$ is continuous and (almost everywhere) twice differentiable with $g'(\cdot) > 0$ and $g''(\cdot) < 0$.

Notice that the first assumption is a normalization. The second one means that the policyholder prefers receiving more money from insurance companies than less and dislikes any mean preserving spread in the net payoff of insurance. As we introduce the notion of loss aversion in Section 4.1, $g(\cdot)$ will contain a kink at the origin and hence will not be fully differentiable everywhere. However, our main results about the optimal insurance demand in Section 3 will be unaffected by this extension. Parameter $k \in [0, +\infty)$, called the degree of narrow framing, measures the weight the policyholder places on the utility from the net payoff of insurance relative to wealth utility. In case of k being zero, the policyholder has standard vNM preferences and only cares about expected wealth utility. In contrast, when the policyholder places a very high weight on expected utility in the net payoff of insurance (i.e., $k \to +\infty$), the insurance purchase becomes hardly different from gambling against the insurer. In more general circumstances, insurance decisions

[9] It is noteworthy that this specification can help to solve Rabin's (2000) paradox: if you turn down a 50-50 gamble of losing \$10 or gaining \$11, and you are an expected utility maximizer, then you should turn down any 50-50 gamble where you may lose \$100, no matter how large the amount you stand to win. Within the standard expected utility framework, non-trivial risk attitudes over small stakes can lead to implausible risk attitudes over large stakes. In our model, $g(\cdot)$ captures risk attitudes over small stakes while utility functions $u(\cdot)$ starts to work when stakes become large.

can be considered as a compromise between two considerations: maximizing expected wealth utility and maximizing expected utility in the net payoff of insurance.[10]

This way of incorporating narrow framing into an otherwise standard expected utility framework was first proposed by Barberis and Huang (2001, 2009). As these authors have argued, narrow framing can also stem from fully rational considerations: for example, from the desire to take non-consumption utility such as regret into account. Regret is the pain we feel when we realize that we would be better off today if we had taken a different action in the past. Even if a gamble that an agent accepts is just one of many risks that he faces, it is still linked to a specific decision, namely the decision to accept the gamble. As a result, it exposes the agent to possible future regret: if the gamble turns out badly, he may regret the decision to accept it. Consideration of non-consumption utility can therefore also lead to preferences that depend directly on the outcomes of specific gambles that the agent faces (Barberis and Huang, 2009, page 1157-1158).

3 The Optimal Insurance Demand

Dating back to as early as the mid-1990s (Johnson et al., 1993), economists have recognized that framing can have a big impact on insurance decisions. For instance, Brown (2007), Brown et al. (2008) and Brown et al. (2011) examined the effects of framing in annuity markets and on claiming of social security benefits. More recently, Gottlieb and Smetters (2016) studied life insurance under narrow framing. In this section, we characterize the optimal insurance-purchasing decision of the policyholder with the preference specified in Equation (2.2) in a very general setting when insurance is available in the form of proportional coverage.[11]

When offered proportional insurance (or coinsurance), the policyholder can insure a fraction α of his initial wealth with $0 \leq \alpha \leq 1$. When a loss x occurs, the policyholder will receive an indemnity of αx. Under linear transaction costs (see Equation (2.1)), the insurance premium is thus given by $(1+\tau)\alpha\mu$ with μ being the expected loss. Given the

[10] An alternative interpretation of our model (2.2) can be that two selves, one being completely rational and the other being a completely narrow framer, are in search of a Pareto-efficient contract.

[11] A straight deductible is another well-known form of insurance contracts. Since similar results would be derived and no further insight can be gained from examining insurance demand when deductible policies are offered, we chose not to include it in the paper.

chosen coverage α and the potential loss x, the final wealth of the policyholder becomes:

$$w(\alpha, x) = w_0 - (1-\alpha)x - (1+\tau)\alpha\mu \tag{3.1}$$

And the net payoff of the purchased coinsurance policy is as follows:

$$h(\alpha, x) = \alpha[x - (1+\tau)\mu] \tag{3.2}$$

The policyholder chooses the optimal coverage α^* such that his objective function specified in Equation (2.2) is maximized. That is,

$$\alpha^* = \arg\max_{\alpha\in[0,1]} V(\alpha) = \arg\max_{\alpha\in[0,1]} \int [u(w(\alpha, x)) + kg(h(\alpha, x))] \, dF(x) \tag{3.3}$$

By differentiating $V(\alpha)$ with respect to α, we obtain:

$$V'(\alpha) = \int u'(w(\alpha, x))\frac{\partial w(\alpha, x)}{\partial \alpha} \, dF(x) + k \int g'(h(\alpha, x))\frac{\partial h(\alpha, x)}{\partial \alpha} \, dF(x) = 0 \tag{3.4}$$

where

$$\frac{\partial w(\alpha, x)}{\partial \alpha} = \frac{\partial h(\alpha, x)}{\partial \alpha} = x - (1+\tau)\mu \tag{3.5}$$

Due to the concavity of utility functions $u(\cdot)$ and $g(\cdot)$, the first-order condition at an interior solution, i.e., $V'(\alpha) = 0$, is both necessary and sufficient for optimality. The first integral in Equation (3.4) is the standard total marginal wealth utility, while the second term is the total marginal utility in the net insurance payoff. Note that a marginal change in the insurance coverage has the same impact on both the final wealth and the net insurance payoff (i.e., $x - (1+\tau)\mu$) (see Equation (3.5)). However, since the marginal wealth utility (i.e., $u'(w(\alpha, x))$) differs from the marginal utility in the net insurance payoff (i.e., $g'(h(\alpha, x))$), the results obtained in the classical EU model will not hold. For instance, we know from Mossin (1968) that in the classical EU model, full insurance is optimal when there is zero deadweight cost. The following proposition states that this is no longer the case under narrow framing.

Proposition 1. *The policyholder with a positive degree of narrow framing (i.e., $k > 0$) strictly prefers partial insurance even when the insurance premium is actuarially fair.*

Proof. See Appendix A. □

The intuition for the above result is straightforward. When insurance is proportional and evaluated in isolation, it can be seen as a pseudo gamble with zero mean in the case of zero loading. Since the policyholder dislikes any zero-mean risks in the net payoff of insurance (i.e., $g''(\cdot) < 0$), there would be no insurance purchase if the role of insurance as hedging the risk to wealth was fully ignored (i.e., $k \to +\infty$). Actually, the policyholder who is a complete narrow framer can be strictly better off by investing into a financial product with the same distribution as the indemnity but at a lower price (e.g., Dybvig, 1988, and Bernard and Vanduffel, 2014).[12] This negative aspect of insurance decreases its demand from the policyholder with narrow framing. In Proposition 1, we only considered the special case where there is zero transaction cost. It is then interesting to compare the optimal coverage proportions implied by the model of narrow framing and by the classical EU model in a more general manner (i.e., $\tau \geq 0$). Note that for a chosen level of insurance coverage α, any increase in the loading factor τ will induce a first-order deterioration in the net insurance payoff (i.e., $I(x) - P$). Since local utility function $g(\cdot)$ is strictly increasing, a policyholder with a positive degree of narrow framing should purchase less coinsurance than a policyholder with zero degree of narrow framing, all other things being equal. This result is summarized in the following proposition.

Proposition 2. *The optimal proportion of insurance coverage α^* defined by Equation (3.3) is decreasing in the degree of narrow framing (i.e., k).*

Proof. See Appendix A. □

As surveyed in Kunreuther and Pauly (2006), there are many different kinds of anomaly in insurance markets which may stem from either the supply side or the demand side. Here, the reason for a low insurance demand falls into their sixth category, namely, *nature of preference formation*. Instead of taking ex-post emotions such as regret (e.g., Bell, 1982; Loomes and Sugden, 1982) or disappointment (e.g., Bell, 1985) into consideration, the policyholder in our model cares about whether he can make money from purchasing insurance, in addition to hedging the risk to wealth.[13] As we can see, narrow framing

[12] It also seems realistic that policyholders consider to invest their premium in the financial market (e.g., Kahane and Nye, 1975; Brockett et al., 2009).

[13] See Braun and Muermann (2004), and Gollier and Muermann (2010) for several interesting applications of regret and disappointment in insurance and finance.

helps to explain the low rate of insurance policy underwriting which is often observed in disaster insurance and health insurance.

4 The Optimal Insurance Design

The pioneering works of Arrow (1971, 1974) explained that a straight deductible policy is the optimal insurance contract for all risk-averse policyholders whenever the insurer's costs are proportional to the indemnity payment and the insurer is risk-neutral. From Proposition 1 in the previous section, we already knew that Arrow's result no longer holds for policyholders with narrow framing at least in the case of zero loading. In the following, we focus on characterizing the shape of function $I(\cdot)$ under narrow framing. In subsection 4.1, we further extend our study to allow for possible loss aversion over the net insurance payoff.

The problem of the policyholder with the preference specified in Equation (2.2) is to select a feasible contract $(I(\cdot), P)$ that maximizes his ex-ante welfare under the insurance tariff constraint given by Equation (2.1):

$$\max_{\{I(.)\geq 0, P\}} \int [u(w_0 - x + I(x) - P) + kg(I(x) - P)]\, dF(x) \tag{4.1}$$

By applying Karush-Kuhn-Tucker (KKT) theorem to the problem, we obtain:

$$\begin{aligned} & L(I(\cdot), P; \lambda, \lambda_x) \qquad (4.2) \\ = & \int \{u(w_0 - x + I(x) - P) + kg(I(x) - P) + \lambda_x I(x) + \lambda[P - (1+\tau)I(x)]\} dF(x) \end{aligned}$$

Parameters λ and λ_x are the Lagrange multipliers associated with the tariff constraint and the non-negativity constraints. Taking a first-order partial derivative with respect to $I(x)$ and P, we have:

$$\frac{\partial L}{\partial I(x)} = [u'(w_0 - x + I(x) - P) + kg'(I(x) - P) + \lambda_x - \lambda(1+\tau)]\, dF(x) = 0 \tag{4.3}$$

$$\frac{\partial L}{\partial P} = \int [-u'(w_0 - x + I(x) - P) - kg'(I(x) - P) + \lambda]\, dF(x) = 0 \tag{4.4}$$

$$\lambda_x I(x) = 0, I(x) \geq 0, \lambda_x \geq 0 \tag{4.5}$$

Or equivalently,

$$u'(w_0 - x + I(x) - P) + kg'(I(x) - P) = \lambda(1 + \tau), \quad \forall x \ s.t \ I(x) > 0 \tag{4.6}$$

$$\int [u'(w_0 - x + I(x) - P) + kg'(I(x) - P)]\, dF(x) = \lambda \tag{4.7}$$

Since L is concave in $(I(\cdot), P)$, the above set of first-order conditions is therefore necessary and sufficient for optimality.

Note that when $k = 0$, Arrow's result applies squarely and a deductible contract is optimal with $I(x) = \max\{0, x - D^*_{EU}\}$ where D^*_{EU} is the unique root of the equation $u'(w_0 - D^*_{EU} - P) = \lambda(1 + \tau)$ with $\int u'(w_0 - x + I(x) - P)\, dF(x) = \lambda$. Namely, for any given premium, the indemnity schedule that maximizes expected wealth utility is that which indemnifies the largest losses in priority. From Mossin (1968), we know that the optimal deductible vanishes when the insurance tariff is fair, and that it is strictly positive when the premium is unfair. Some of these results can be generalized to the case of narrow framing but others cannot. This is summarized in the following proposition.

Proposition 3. *Under the preference described in Equation (2.2) with $k \geq 0$, the optimal contract $(I(\cdot), P)$ is such that there is a deductible $D^* \geq 0$ and*
(i) When $\tau = 0$, the optimal deductible level D^* should be set equal to zero. When $\tau > 0$, D^* is strictly positive.
(ii) The marginal coverage satisfies that for all x such that $I(x) > 0$,

$$\frac{\partial I(x)}{\partial x} = \frac{u''(w_0 - x + I(x) - P)}{u''(w_0 - x + I(x) - P) + kg''(I(x) - P)} \in [0, 1] \tag{4.8}$$

(iii) The marginal coverage is increasing, i.e., $\partial^2 I(x)/\partial x^2 \geq 0$, if $u'''(\cdot) \geq 0$ and $g'''(\cdot) \geq 0$.

Proof. See Appendix A. □

The first point in Proposition 3 shares the same idea as that in straight deductible contracts, i.e., that when insurance premium is not actuarially fair, it is not optimal to insure people in all states of losses. Intuitively, if a positive indemnity was paid in all states, a small uniform reduction in the indemnity would increase wealth in all states because the reduction in the premium would be larger than the reduction in indemnity. Under standard expected utility theory, the classical result of Arrow (1971, 1974) showed that these states with no indemnity are for low losses. This result remains true under narrow framing.

The second point in Proposition 3 states that the net loss $(x - I(x))$ incurred by the policyholder is increasing with the loss in the loss domain in which an indemnity is paid. Namely, the optimal contract contains a deductible and coinsurance of losses above the deductible. From Equation (4.6), we can see that marginal utility at all insured states consists of two parts: marginal wealth utility and marginal utility from the net payoff of insurance $(I(x) - P)$, and at the optimum it must be equalized. Otherwise, expected utility can be increased by moving an infinitesimal amount of indemnity from a state with low marginal utility to a state with high marginal utility without changing the insurance premium. Since marginal wealth utility is higher at a higher loss, intuitively, it is optimal to compensate the insured individual more when he faces a higher loss. However, as the indemnity increases, the marginal utility from the net payoff of insurance becomes smaller. In order to maintain the marginal utility constant, the indemnity should be increased relatively less compared to loss.

The last point in Proposition 3 is closely related to the study of precautionary saving (Kimball, 1990). As both $u'(\cdot)$ and $g'(\cdot)$ are convex,[14] that is, marginal utilities in wealth and the net payoff of insurance decrease at a higher rate as losses increase, it is prudent to compensate the insured at a higher rate when his loss is higher.

To illustrate our obtained results so far, we consider the following numerical example. Let us suppose that the policyholder has an initial wealth $w_0 = 10$ and faces a risk of loss $x \in \{0, 1, ..., 9\}$ with all loss states being uniformly distributed. The policyholder has the preference given by Equation (2.2) with $k = 0.1$, $u(w) = \sqrt{w}$ and $g(w) = -exp(-w)$. The loading factor τ equals 5%. These specifications satisfy all the assumptions we made previously. We solved the problem numerically. Under zero narrow framing (i.e., $k = 0$), the optimal deductible is $D^*_{EU} = 3.03$, and the optimal contract is depicted by the plain curve in Figure 1. The dots in Figure 1 describe the optimal contract when the policyholder has a degree of narrow framing k equal to 0.1. For losses above $D^* = 4.94$, the indemnity is strictly increasing in losses but the marginal coverage is strictly less than

[14] Note that hyperbolic absolute risk aversion (HARA) utility functions except quadratic utility functions, which are widely adopted in economics, exhibit a positive-third derivative.

FIGURE 1. Optimal Contract under Narrow Framing

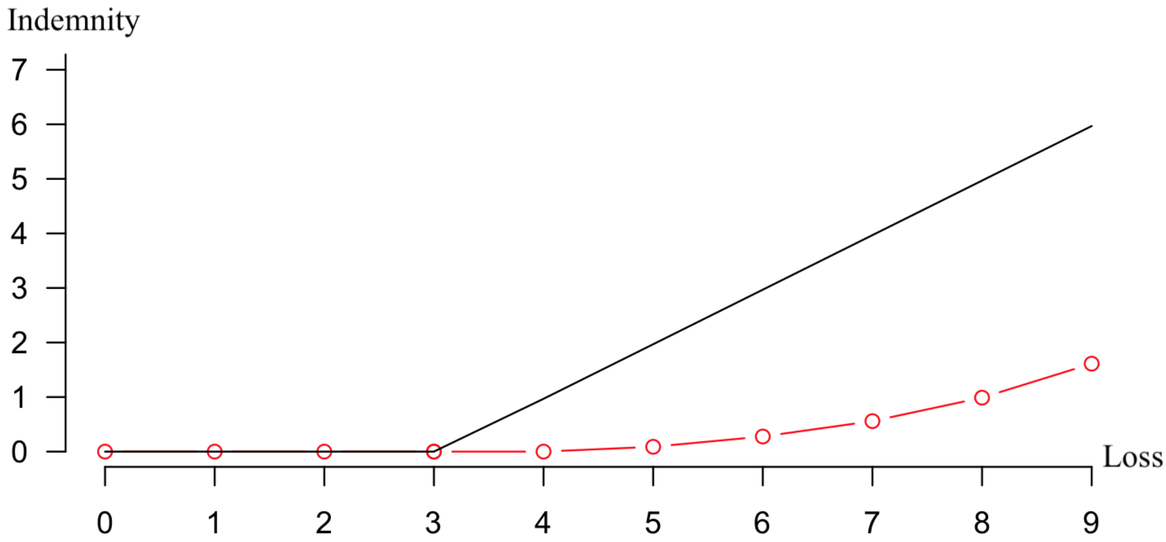

Figure notes: the support of loss is $\{0, 1, 2, ..., 9\}$. All loss states are equally likely. We also assume total expected utility $V(I(\cdot), P) = \int [u(w_0 - x + I(x) - P) + kg(I(x) - P)]\, dF(x)$ with $k = 0.1$ being the degree of narrow framing, $v(w) = \sqrt{w}$ and $g(w) = -exp(-w)$. The loading factor τ equals 5%. The dots correspond to the optimal insurance schedule under narrow framing, whereas the plain curve is the optimal insurance schedule if $k = 0$.

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We further consider a special case where both functions $u(\cdot)$ and $g(\cdot)$ are quadratic, i.e., $u(w) = w - b_u w^2$ and $g(w) = w - b_g w^2$ with $2w_0 \leq \min\{1/b_u, 1/b_g\}$ and b_u, $b_g > 0$.[15] From the functional form of the marginal coverage in Proposition 3 (see Equation (4.8)), we know that the marginal coverage is constant and equal to $b_u/(b_u + kb_g)$. The following corollary summarizes these results.

Corollary 1. *When the policyholder with the preference specified in Equation (2.2) uses quadratic functions to evaluate wealth utility and the utility of the net payoff of insurance, i.e,* $u(w) = w - b_u w^2$ *and* $g(w) = w - b_g w^2$ *with* $2w_0 \leq \min\{1/b_u, 1/b_g\}$ *and* b_u, $b_g > 0$, *a*

[15] In financial economics, the quadratic utility function is the most frequently used to describe an investor's behavior. Under the assumption of quadratic utility, mean-variance analysis of modern portfolio theory by Markowitz (1959) is optimal for any distribution of risk. Moreover, functional magnetic resonance imaging studies of the brain suggest that the mean-variance approach is consistent with brain functions (e.g., Preuschoff et al., 2008).

FIGURE 2. Optimal Contract under Narrow Framing with Quadratic Utilities

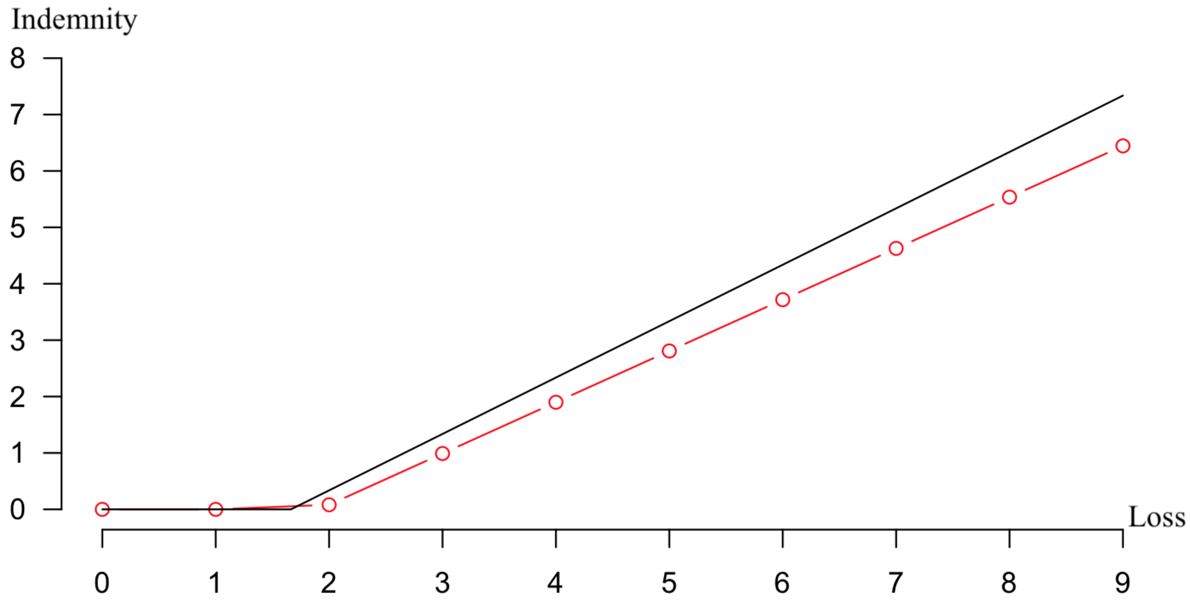

Figure notes: the support of loss is $\{0, 1, 2, ..., 9\}$. All loss states are equally likely. We also assume total expected utility $V(I(\cdot), P) = \int [u(w_0 - x + I(x) - P) + kg(I(x) - P)]\, dF(x)$ with $k = 0.1$ being the degree of narrow framing and $v(w) = g(w) = w - 0.05w^2$. The loading factor τ equals 5%. The dots correspond to the optimal insurance schedule under narrow framing, whereas the plain curve is the optimal contract if $k = 0$.

proportional deductible contract is optimal, i.e.,

$$I(x) = \frac{b_u}{b_u + kb_g} \max\{0, x - D^*\} \tag{4.9}$$

When $\tau = 0$, the optimal deductible level D^* should be set equal to zero. When $\tau > 0$, D^* is strictly positive.

As the degree of narrow framing increases, i.e., the policyholder puts more weight on utility of the net of insurance payoff, the optimal coinsurance rate (i.e., $b_u/(b_u + kb_g)$) decreases. This is in line with Proposition 1 that the policyholder chooses to be partially insured when offered coinsurance contracts with zero transaction cost. It is noteworthy that when the transaction cost is zero, the optimal insurance contract can be simply created by dividing the insurance premium and the indemnity of the full insurance contract by $b_u/(b_u + kb_g)$.

In Figure 2, we illustrate Proposition 1 with a numerical example. With everything else kept as in our previous example, we changed the utility functions to be quadratic.

More specifically, $v(w) = g(w) = w - 0.05w^2$. Under zero narrow framing (i.e., $k = 0$), the optimal deductible is $D^*_{EU} = 1.66$, and the optimal insurance contract is depicted by the plain curve in Figure 2. The dots in Figure 2 describe the optimal insurance contract when the policyholder has a degree of narrow framing k equal to 0.1. For a loss above $D^* = 1.91$, indemnity is strictly increasing in losses but the marginal coverage is strictly less than 1 and equal to 10/11.

4.1 Narrow Framing with Loss Aversion

So far, we have assumed that utility function $g(\cdot)$ is twice differentiable everywhere. In this section, we relax this assumption to take into account possible loss aversion (Kahneman and Tversky, 1979) that losses loom larger than gains or mathematically, $\lim_{\Delta\to 0^-} g'(\Delta) > \lim_{\Delta\to 0^+} g'(\Delta)$.[16] To do so,[17] we introduce a kink at the break-even point where the insurance indemnity is equal to the insurance premium and assume that the local gain-loss utility function $g(\cdot)$ has a piecewise linear form as follows:

$$g(I(x) - P) = \begin{cases} I(x) - P, & \text{if } I(x) \geq P; \\ -\beta[P - I(x)], & \text{if } I(x) < P. \end{cases} \tag{4.10}$$

where $\beta > 1$ measures the degree of loss aversion.[18] Note that $g(\cdot)$ is still globally concave. Thus, our main results in Section 3 remain valid. This specification of function $g(\cdot)$ also implies that the policyholder is first-order risk averse in the net payoff of insurance, as defined in Segal and Spivak (1990), at the break-even point. In the following proposition, we present our result on the optimal insurance contract under narrow framing with loss aversion.

[16] We refer to Köbberling and Wakker (2005) for more details on the index of loss aversion.

[17] Similar results can be obtained if we assume function $g(\cdot)$ is concave in both the gain and loss domains, instead of being linear.

[18] The same specification has been widely adopted in the literature on finance. See, for instance, Barberis and Huang (2001, 2009). We abstract ourselves from the possibility of risk seeking in the loss domain of the net payoff of insurance, i.e., $g''(x) > 0$ for $x < 0$ (Kahneman and Tversky, 1979). Adding this convexity considerably complicates our analysis but does not add more interesting insights. Moreover, as many experimental studies have shown, a piece-wise linear function is a good approximation. For instance, assuming a power function for the value function for gains and for losses, Tversky and Kahneman (1992) found a median value of 0.88 for the power coefficient for both gains and losses, and Abdellaoui et al. (2007) found a median value of 0.86 for the power coefficient for gains and a median value of 0.80 for the power coefficient for losses.

FIGURE 3. Graphical Illustration of the Optimal Insurance Scheme with Loss Aversion

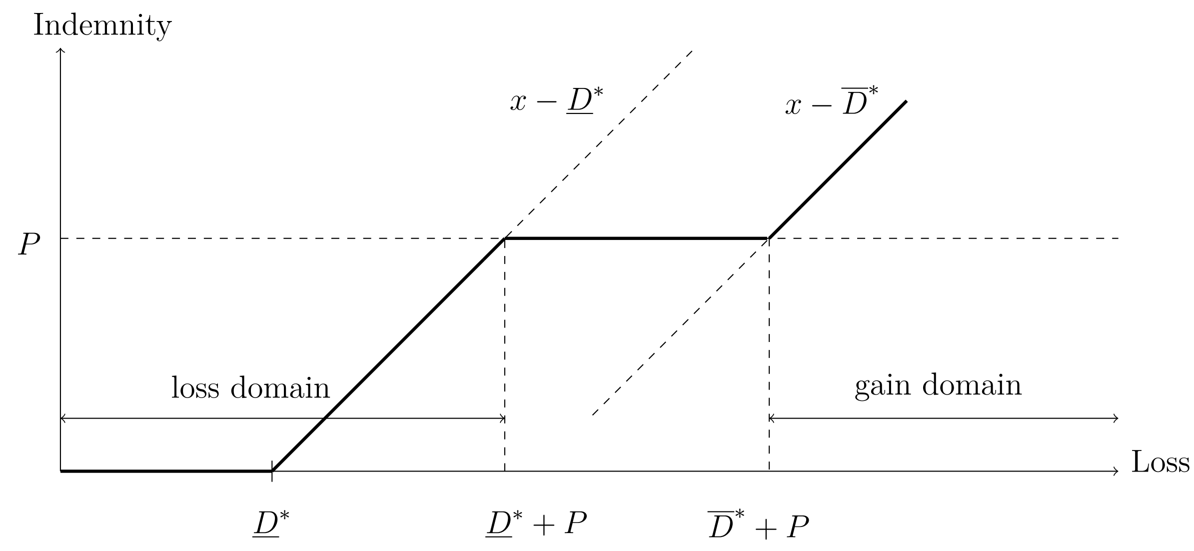

Figure notes: the thick line represents the optimal insurance scheme for a gain-loss function $g(\cdot)$ specified in Equation (4.10).

Proposition 4. *Under the preference described in Equation (2.2) with $k \geq 0$ and a gain-loss utility $g(\cdot)$ as in Equation (4.10), it is optimal to have the following indemnity function:*

$$I(x) = \begin{cases} 0 & \text{if } x \leq \underline{D}^* \\ x - \underline{D}^* & \text{if } \underline{D}^* < x < \underline{D} + P \\ P & \text{if } \underline{D}^* + P \leq x \leq \overline{D}^* + P \\ x - \overline{D}^* & \text{if } x > \overline{D}^* + P \end{cases} \tag{4.11}$$

with $\overline{D}^* \geq \underline{D}^* \geq 0$. When $\tau = 0$, it is optimal to be insured in all states of losses, i.e., $\underline{D}^* = 0$. When $\tau > 0$, $\underline{D}^*$ is strictly positive.

Proof. See Appendix A. □

It is noteworthy that the optimal insurance scheme in Proposition 4 can also be represented as $I(x) = \max\left\{0, \min\{x - \underline{D}^*, P\}, x - \overline{D}^*\right\}$ with $\overline{D}^* \geq \underline{D}^*$. Thus, it has a feature of straight deductible contracts for both low and high losses but pays a fixed amount of indemnity equal to the premium for a range of intermediate losses. Figure 3 illustrates the optimal insurance scheme under narrow framing with loss aversion. The intuition behind this result is that since losses loom larger than gains, it is optimal to compensate the insured individual relatively more in the loss domain where x is small but less in the gain domain where x is large (i.e., the line $x - \underline{D}^*$ lying strictly above the line $x - \overline{D}^*$ in Figure 3) in order to reduce the range of the loss domain. A flat payment scheme for intermediate values of x shares the same idea. Moreover, as long as there is no transaction cost, it is optimal to be insured in all possible states, i.e. $\underline{D}^* = 0$. Finally, it is noteworthy that the net loss $(x - I(x))$ that the insured individual undergoes is increasing in loss similar to what we obtained previously.

To further illustrate this result, we numerically solved the optimization problem of the policyholder with the same calibration as in Figure 1, except for function $g(\cdot)$. Here we changed $g(\cdot)$ to be piecewise linear with $g(x) = x$ for $x \geq 0$ and $g(x) = 2x$ for $x < 0$. With a degree of narrow framing k equal to 0.1, the optimal insurance contains two deductibles with $\underline{D}^* = 4.68$ and $\overline{D}^* = 7.26$ for low losses and high losses. For any intermediate losses between 5.05 and 7.63, a fixed indemnity equal to the premium with $P = 0.37$ is paid.

5 Discussions and Conclusion

We have shown that our model of narrow framing could explain the low insurance demand observed in many insurance markets such as health insurance and disaster insurance. However, there are also situations where people insure themselves too much (Sydnor, 2010). This may be because the reference point is shifted when moving from one decision situation to another. A recent work by Eeckhoudt et al. (2018) studied insurance purchase from the policyholder who is first-order risk averse, as defined by Segal and Spivak (1990), at the full insurance wealth level which plays a role of reference point or target. Not reaching this target is painful for the policyholder. They showed that full insurance can be optimal even with a positive premium loading. More research works on what determines the formation of the reference point would be therefore desirable.

In our model we have also abstracted ourselves from possible risk seeking behavior in the loss domain and probability weighting as Tversky and Kahneman (1992)'s prospect theory would suggest. It would be of course interesting to take into account all these elements. However, this would much complicate our analysis and lose tractability. Our main objective in this paper is to examine how narrow framing (or possibly joint with loss aversion) affects insurance decisions and the design of an optimal insurance policy. We believe that our model is not only parsimonious but also rich enough to serve our purposes.

As a final remark, we have assumed that the policyholder does not have any control over the distribution of losses. Since the insured under narrow framing cares about whether insurance brings him money or not, he may have less incentive to exert effort in prevention and may even be willing to take more risks. This should be distinguished from the classical moral hazard problem. In our model, exerting less effort or taking more risks is driven by the motive of recovering as much as possible the insurance premium paid to the insurer. To address agency problems and its feedback on contracting is, however, left for future research.

To conclude, in this paper we examined the optimal insurance demand and design of insurance contracts under narrow framing. The policyholder in our model has mixed views with regard to insurance purchases: hedging the risk to wealth and gambling against insurance companies. Since the net insurance payoff is perceived as an unfavorable gamble in isolation by the policyholder, partial insurance is strictly preferred by the latter even when there is zero transaction cost. This helps us to understand the low insurance demand observed in many insurance markets. The optimal insurance design under narrow framing (with possible loss aversion) was also characterized. It turns out that straight deductible contracts as suggested by Arrow are not optimal anymore. When considering narrow framing alone, the optimal contract should contain a deductible and coinsurance of losses above it. Although these contractual features were shown to be related to risk preferences and cost functions of the insurer or insurance fraud, we provided an alternative explanation for why it may be optimal. As the policyholder is loss averse over the net insurance payoff, a flat payment of the insurance premium should be applied to a range of intermediate losses. In the end, we would like to emphasize the importance of combining traditional insurance economics with behavioral economics. This generates new insights to explain actual behaviors.

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Appendix A: Proofs

Proof of Proposition 1 and 2

Here we prove Proposition 1 and 2 at the same time. First of all, we rewrite the second integral in Equation (3.4) as follows:

$$\begin{aligned}
&\int g'(h(\alpha,x))\frac{\partial h(\alpha,x)}{\partial\alpha}\,dF(x)\\
=&\int g'(\alpha(x-(1+\tau)\mu))[x-(1+\tau)\mu]\,dF(x)\\
=&\int_0^{[(1+\tau)\mu]^-} g'(\alpha(x-(1+\tau)\mu))[x-(1+\tau)\mu]\,dF(x)\\
&+0\times g'(0)\times P_x((1+\tau)\mu)\\
&+\int_{[(1+\tau)\mu]^+}^{w_0} g'(\alpha(x-(1+\tau)\mu))[x-(1+\tau)\mu]\,dF(x)
\end{aligned} \tag{A.1}$$

where $P_x((1+\tau)\mu) = \text{Prob}(x=(1+\tau)\mu) \geq 0$.
The concavity of local utility function $g(\cdot)$, i.e., $g''(\cdot)<0$, implies that

$$g'(0) > g'(\alpha(x-(1+\tau)\mu)), \quad \forall x > (1+\tau)\mu \tag{A.2}$$

Applying this relationship to Equation (A.1), we achieve the following inequality:

$$\begin{aligned}
&\int g'(h(\alpha,x))\frac{\partial h(\alpha,x)}{\partial\alpha}\,dF(x)\\
<&\int_0^{[(1+\tau)\mu]^-} g'(\alpha(x-(1+\tau)\mu))[x-(1+\tau)\mu]\,dF(x)\\
&+0\times g'(0)\times P_x((1+\tau)\mu)\\
&+g'(0)\int_{[(1+\tau)\mu]^+}^{w_0}[x-(1+\tau)\mu]\,dF(x)\\
=&g'(0)\Big\{\int_0^{[(1+\tau)\mu]^-}\frac{g'(\alpha(x-(1+\tau)\mu))}{g'(0)}[x-(1+\tau)\mu]\,dF(x)\\
&+0\times P_x((1+\tau)\mu)\\
&+\int_{[(1+\tau)\mu]^+}^{w_0}[x-(1+\tau)\mu]\,dF(x)\Big\}
\end{aligned} \tag{A.3}$$

Again since $g''(\cdot) < 0$, we have:

$$\frac{g'(\alpha(x-(1+\tau)\mu))}{g'(0)} > 1, \quad \forall x < (1+\tau)\mu \tag{A.4}$$

Finally, this allows us to write the inequality (A.3) as follows:

$$\begin{aligned}\int g'(h(\alpha,x))\frac{\partial h(\alpha,x)}{\partial \alpha}\,dF(x) < g'(0)\Big\{ &\int_0^{[(1+\tau)\mu]^-} [x-(1+\tau)\mu]\,dF(x) \\ &+ 0 \times P_x((1+\tau)\mu) \\ &+ \int_{[(1+\tau)\mu]^+}^{w_0} [x-(1+\tau)\mu]\,dF(x)\Big\} \\ &= -\tau\mu g'(0)\ (\leq 0)\end{aligned} \tag{A.5}$$

Due to the concavity of function $g(\cdot)$, the second integral in Equation (3.4) is strictly negative for any $\tau \geq 0$. Therefore, $V'(\cdot)$ is strictly decreasing in k. This further implies that a policyholder with a higher degree of narrow framing will purchase less insurance, ceteris paribus. In particular, as $\tau = 0$, the policyholder with zero degree of narrow framing (i.e., $k = 0$) has vNM expected utility and prefers full insurance when (Mossin 1968). But the policyholder with a positive degree of narrow framing (i.e., $k > 0$) will choose to be partially insured.

Proof of Proposition 3

We begin with points (ii) and (iii) of Proposition 3 then its point (i).

Point (ii): differentiating Equation (4.6) with respect to x, we get:

$$u''(w_0 - x + I(x) - P)(-1 + \frac{\partial I(x)}{\partial x}) + kg''(I(x) - P)\frac{\partial I(x)}{\partial x} = 0, \ \forall x \ s.t \ I(x) > 0 \tag{A.6}$$

Rearranging it gives us:

$$\frac{\partial I(x)}{\partial x} = \frac{u''(w_0 - x + I(x) - P)}{u''(w_0 - x + I(x) - P) + kg''(I(x) - P)}, \ \forall x \ s.t \ I(x) > 0 \tag{A.7}$$

Under the assumptions of concavity (i.e., $u''(\cdot) < 0$ and $g''(\cdot) < 0$), it is easy to check that $0 \leq \partial I(x)/\partial x \leq 1$ for any $k \geq 0$.

Point (iii): taking a first-order derivative with respect to x in Equation (A.6), we obtain

$$\frac{\partial^2 I(x)}{\partial x^2} = \frac{-kg'''(u'' + kg'')\frac{\partial I(x)}{\partial x} + kg''[-u''' + (u''' + kg''')\frac{\partial I(x)}{\partial x}]}{(u'' + kg'')^2} \tag{A.8}$$

Notice that $\partial^2 I(x)/\partial x^2$ has the same sign as its nominator. Rearranging the nominator gives

$$\begin{aligned} & -kg'''(u'' + kg'')\frac{\partial I(x)}{\partial x} + kg''[-u''' + (u''' + kg''')\frac{\partial I(x)}{\partial x}] \\ = & -kg'''u''\frac{\partial I(x)}{\partial x} - kg''u''' + kg''u'''\frac{\partial I(x)}{\partial x} \\ = & -k[g'''u''\frac{\partial I(x)}{\partial x} + (1 - \frac{\partial I(x)}{\partial x})g''u'''] \end{aligned} \tag{A.9}$$

From point (ii) of the proposition, we know that $0 < \partial I(x)/\partial x < 1$. Therefore, assuming that $u'''(\cdot) \geq 0$ and $g'''(\cdot) \geq 0$, we can demonstrate that the nominator is non-negative. This directly implies that $\partial^2 I(x)/\partial x^2 \geq 0$.

Point (i): By continuity of functions $u(\cdot)$ and $g(\cdot)$, $I(x)$ should also be continuous over $[0, w_0]$. From point (ii) of the proposition, we know that $I(x)$ is increasing when it's positive. So there must exist a constant $D^* \geq 0$ such that $I(x) = 0$ for $x \leq D^*$ but $\lim_{x \to D^{*+}} I(x) > 0$. Moreover, we know that D^* is the solution to the equation

$$u'(w_0 - D^* - P) + kg'(-P) = \lambda(1 + \tau) \tag{A.10}$$

When $\tau = 0$, combining the above equation with Equation (4.7) gives:

$$\begin{aligned} & u'(w_0 - D^* - P) + kg'(-P) \\ = & \int_0^{D^*} [u'(w_0 - x - P) + kg'(-P)]\, dF(x) + \\ & \int_{D^*}^{w_0} [u'\big(w_0 - D^* - P\big) + kg'\big(-P\big)]\, dF(x) \end{aligned} \tag{A.11}$$

Because functions $u'(\cdot)$ is strictly decreasing in x (i.e., $u''(\cdot) < 0$), Equation (A.11) is true if and only if $D^* = 0$. When τ is positive, suppose by contradiction that $I(x)$ is positive almost surely (using probability measure F). This implies that Equation (4.6) is satisfied almost surely for any x. Integrating Equation (4.6) with respect to x yields:

$$\int u'(w_0 - x + I(x) - P) + kg'(I(x) - P)\, dF(x) = \lambda(1 + \tau) \tag{A.12}$$

Because $\tau > 0$, this is in contradiction with Equation (4.7). Thus, there must exist a subset of losses of positive measure (under F) such that $I(x) = 0$. Since $I(x)$ is continuous and increasing over $[0, w_0]$, it must be that $D^* > 0$.

Proof of Proposition 4

The proof here is quite similar to that of Proposition 3. We essentially make use of the fact that marginal utility at all insured states must be equalized at optimum. Differentiating $V(I(\cdot), P)$ with respect to $I(x)$ when it is positive at optimum, we obtain

$$u'(w_0 - x + I(x) - P) + k = \lambda(1 + \tau), \quad \text{if } I(x) - P > 0 \tag{A.13}$$

and

$$u'(w_0 - x + I(x) - P) + k\beta = \lambda(1 + \tau), \quad \text{if } -P \leq I(x) - P < 0 \tag{A.14}$$

where λ represents the marginal utility increase following one unit decrease in insurance premium at optimum. Let us define $\overline{D}^*$ and $\underline{D}^*$ in such a way that

$$u'(w_0 - \overline{D}^* - P) + k = \lambda(1 + \tau), \quad \text{if } I(x) - P > 0 \tag{A.15}$$

and

$$u'(w_0 - \underline{D}^* - P) + k\beta = \lambda(1 + \tau), \quad \text{if } -P \leq I(x) - P < 0 \tag{A.16}$$

Obviously, because of the concavity of function $u(\cdot)$, Equation (A.13) and (A.14) are satisfied with

$$I(x) = x - \overline{D}^*, \quad \text{if } x > \overline{D}^* + P \tag{A.17}$$

and

$$I(x) = x - \underline{D}^*, \quad \text{if } \underline{D}^* \leq x < \underline{D}^* + P \tag{A.18}$$

Combining Equation (A.15) with (A.16), we obtain:

$$u'(w_0 - \overline{D}^* - P) - u'(w_0 - \underline{D}^* - P) = (\beta - 1)k \geq 0 \tag{A.19}$$

The last inequality is due to loss aversion, i.e., $\beta > 1$. Again because $u''(\cdot) < 0$, we can show that $\overline{D}^*$ is no less than $\underline{D}^*$. Note that $I(x)$ should be continuous and non-decreasing because function $u(\cdot)$ and $g(\cdot)$ are continuous and concave. This implies that

$$I(x) = P, \quad \forall x \in [\underline{D}^* + P, \overline{D}^* + P] \tag{A.20}$$

Namely, $I(x)$ is equal to the insurance premium for any losses between $\underline{D}^* + P$ and $\overline{D}^* + P$. Since marginal utility at all insured states must be equalized at optimum and by the definition of λ, we're allowed to write the following equation:

$$\int_0^{\underline{D}^*} [u'(w_0 - x - P) + kg'(-P)] \, dF(x) + \lambda(1+\tau) \int_{\underline{D}^*}^{w_0} dF(x) = \lambda \tag{A.21}$$

Following the same reasoning as in the proof of Proposition 3, we can show that $\underline{D}^*$ must be set to zero as $\tau = 0$ and that $\underline{D}^*$ must be positive as $\tau > 0$.